Louisiana Mu Alpha Theta

affiliated with
Mu Alpha Theta
National High School and Junior College
Honorary Mathematics Club

50th State Convention
March 25-27, 2010
Baton Rouge, Louisiana

Mu B - Area

TEST RULES

1. Do not begin test until you are told to do so.
2. You must supply your own #2 pencil.
3. Only ACT approved calculators are allowed on all tests.
4. Print your name and school in the name blank, your code in the date blank, and the area test in the subject blank on your Scantron answer sheet
5. Standard procedure for machine graded papers must be followed. Use only a #2 pencil, marking the appropriate spaces carefully.
6. In case of a tie, winners will be determined according to the order in which the answer sheet was turned into the moderator.
7. Do all scratch work on your test.

1. In the xy plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$ for $-3 \le t \le 3$ is a line segment with slope

a. $\frac{3}{5}$ b. $\frac{5}{3}$ c. 3 d. 5 e. NOTA

2. Which of the following does not have an antiderivative?

a. $\ln(x)$ b. e^{x^2} c. $\frac{1}{1+x^2}$ d. $\cos^2(x)$ e. NOTA

3. When r is a real number greater than 1

$$\int_1^r \frac{1}{x\ln(x)} =$$

a. 0 b. ∞ c. $\ln(r)$ d. $-\infty$ e. NOTA

4. Find the total length of the closed curve with equation $r = 2a\cos\theta$ for $a < 0$

a. $4\pi a$ b. πa c. $2\pi a$ d. $4\pi a$-1 e. NOTA

5. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

a. $\int_0^1 \sqrt{t^2+1}\,dt$ b. $\int_0^1 \sqrt{t^2+t}\,dt$ c. $\int_0^1 \sqrt{t^4+t^2}\,dt$ d. $\frac{1}{2}\int_0^1 \sqrt{4+t^4}\,dt$ e. NOTA

6. Evaluate the limit as x goes to one of $\int_1^x \frac{e^{t^2}}{x^2 - 1}\, dt$

a. 0 b. 1 c. $\frac{e}{2}$ d. e e. NOTA

7. What is the approximation of the value of sin (1) obtained by using the fifth-degree Taylor polynomial about x = 0 for sin(x)

a. $1 - \frac{1}{2} - \frac{1}{24}$ b. $1 - \frac{1}{6} + \frac{1}{20}$ c. $1 - \frac{1}{2} + \frac{1}{4}$ d. $1 - \frac{1}{3} + \frac{1}{5}$ e. NOTA

8. Find the area of the region that lies inside $r = 3sin\theta$ and outside $r = 1 + sin\theta$

a. $\frac{3\sqrt{3}}{2}$ b. 2π c. $\frac{3\sqrt{3}}{4}$ d. $\frac{3\pi}{4}$ e. NOTA

9. Evaluate $\int_0^1 arctan(x)\, dx$

a. $\frac{\pi}{2} - ln(\sqrt{2})$ b. $\frac{\pi}{2} - ln(2)$ c. $\frac{1}{2}(\pi - ln(\sqrt{2}))$ d. $\frac{1}{2}(\pi - ln(2))$ e. NOTA

10. Given $y = cosh(x)$, find the arc length from (0,1) to (1,cosh(1))

a. $\frac{e + e^{-1}}{2}$ b. $\frac{1}{2}e - e^{-1}$ c. $\frac{e - e^{-1}}{4}$ d. $\frac{e - e^{-1}}{2}$ e. NOTA

11. Evaluate $\int_{-2}^{2} (3[x] - 2|x|)\, dx$ where [x] represents the greatest integer less than or equal to x.

a. -4 b. -16 c. -10 d. -7 e. NOTA

12. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

a. $-3 < x \le 3$ b. $-3 \le x \le 3$ c. $-2 < x < 4$ d. $-2 \le x < 4$ e. NOTA

13. Which of the following is the trapezoidal approximation for, $f(x) = \frac{1}{x}$ on the interval [1,3] with n equal subintervals?

a. $\frac{1}{n}\left[1 + \frac{2}{1+\frac{2}{n}} + \frac{2}{1+\frac{4}{n}} + \frac{2}{1+\frac{6}{n}} + \ldots + \frac{2}{1+\frac{2(n-1)}{n}} + \frac{1}{3}\right]$

b. $\frac{2}{n}\left[1 + \frac{2}{1+\frac{2}{n}} + \frac{2}{1+\frac{4}{n}} + \frac{2}{1+\frac{6}{n}} + \ldots + \frac{2}{1+\frac{2(n-1)}{n}} + \frac{1}{3}\right]$

c. $\frac{2}{n}\left[1 + \frac{2}{1+\frac{1}{n}} + \frac{2}{2+\frac{2}{n}} + \frac{2}{1+\frac{3}{n}} + \ldots + \frac{2}{1+\frac{(n-1)}{n}} + \frac{1}{3}\right]$

d. $\frac{1}{n}\left[1 + \frac{2}{1+\frac{1}{n}} + \frac{2}{2+\frac{2}{n}} + \frac{2}{1+\frac{3}{n}} + \ldots + \frac{2}{1+\frac{(n-1)}{n}} + \frac{1}{3}\right]$

e. NOTA

14. What is the volume generated when the region satisfying $2x < y < 3x$ and $3 < x < 5$ is rotated about the y-axis?

a. $\frac{157\pi}{5}$ b. $\frac{169\pi}{5}$ c. $\frac{182\pi}{5}$ d. $\frac{196\pi}{5}$ e. NOTA

15. Evaluate $\frac{d}{dt}\left(\int_{6t}^{1} (1+\sqrt{x})^2\, dx\right)$

a. $\sqrt{1+6t}$ b. $-(1+\sqrt{6t})^2$ c. $(1+\sqrt{6t})^2$ d. $-6(1+\sqrt{6t})^2$ e. NOTA

16. Which of the following is true for $f(x) = cos(x)$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ using four equal subintervals?

a. left < right b. right < left c. mid < left d. left = right e. NOTA

17. Evaluate the indefinite integral $\int \frac{1}{x^2 + 2x - 8}\, dx$

a. $ln|x^2 + 2x - 8| + C$ b. $\frac{1}{6} ln\left|\frac{x-2}{x+4}\right| + C$ c. $\frac{1}{3} arctan\left(\frac{x+1}{3}\right) + C$

d. $-\frac{1}{3} arctan\left(\frac{x+1}{-3}\right) + C$ e. NOTA

18. Find the ordered pair (a,b) such that $\int_a^b (10y - 16 - y^2)\, dy$ is a maximum

a. (4,8) b. (2,8) c. (0,4) d. (4,10) e. NOTA

19. What is the area of one of the petals of the polar graph $r = 2sin(4\theta)$

a. $\frac{\pi}{8}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$ e. NOTA

20. Which of the following improper integrals converges to a defined value?

a. $\int_0^\infty xe^{-x}\, dx$ b. $\int_0^\infty \frac{1}{x^3}\, dx$ c. $\int_1^\infty \frac{1}{x}\, dx$ d. $\int_0^\infty cos(\pi x)\, dx$ e. NOTA

21. A tangent line can be drawn on the polar graph $r = 3 - 3cos(\theta)$ such that the line is tangent to exactly two distinct points on the graph. What are the polar coordinates of these two points?

a. $\left(\frac{9}{2},\frac{2\pi}{3}\right)$ & $\left(\frac{9}{2},\frac{4\pi}{3}\right)$ b. $\left(1{,}3-\frac{3\sqrt{3}}{2}\right)$ & $\left(1{,}3+\frac{3\sqrt{3}}{2}\right)$ c. $(6,\pi)$ & $(0,0)$

d. $\left(\frac{3}{2},\frac{\pi}{3}\right)$ & $\left(\frac{3}{2},\frac{5\pi}{3}\right)$ e. NOTA

22. What is the surface area of the object formed by rotating the circle $(x-3)^2 + y^2 = 1$ around the y-axis?

a. $6\pi^2$ b. 6π c. 12π d. $12\pi^2$ e. NOTA

23. Find the limit as x goes to zero of $\frac{x}{3\sin(x)\cos(x)}$

a. $\frac{1}{2}$ b. $\frac{1}{6}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. NOTA

24. Evaluate the indefinite integral $\int \sqrt{4x^2+20}\,dx$

a. $\sqrt{x^2+5}+5\ln\left|\sqrt{x^2+5}\right|+C$ b. $x\sqrt{x^2+5}+\ln\left|\sqrt{x^2+5}+x\right|+C$

c. $\sqrt{x^2+5}+\ln\left|\sqrt{x^2+5}+x\right|+C$ d. $x\sqrt{x^2+5}+\ln\left|\sqrt{x^2+5}+x\right|+C$ e. NOTA

25. Evaluate the indefinite integral $\int \frac{e^x - e^{-x}}{2}$

a. $e^{\frac{x}{2}}+e^{-\frac{x}{2}}+C$ b. $e^{\frac{x}{2}}-e^{-\frac{x}{2}}+C$ c. $\frac{e^x - e^{-x}}{2}+C$ d. $e^{x^2}+e^{-x^2}+C$ e. NOTA

Tie Breaker: Find the positive value of k for which $\int_0^{\frac{\pi}{2}} |\cos(x) - kx|$ is minimized