



Louisiana Mu Alpha Theta

affiliated with
Mu Alpha Theta
National High School and Junior College
Honorary Mathematics Club

50th State Convention March 25-27, 2010 Baton Rouge, Louisiana

Interschool Theta

TEST RULES

1. Do not begin test until you are told to do so.
2. You must supply your own #2 pencil.
3. Only ACT approved calculators are allowed on all tests.
4. Print your name, school, and your code on your answer sheet.
5. In case of a tie, winners will be determined according to the order in which the answer sheet was turned into the moderator.
6. Do all scratch work on your test.

Lower Interschool

1. What symbol can one place between 2 and 3 in order to achieve a number greater than two but less than three?
2. What is the last digit of 7^{1732} ?
3. What is the maximum number of intersections that can be formed by seven coplanar lines?
4. A circle is inscribed inside of a square. The distance from the center of the circle to a vertex of a square is $\frac{\sqrt{2}}{2}$. What is the area of the square that is not occupied by the circle?
5. Simplify i^{2010} , where $i = \sqrt{-1}$.
6. How many primes are there less than 100?
7. The sum of 27 consecutive integers is 1107. What is the smallest of these integers?
8. Either Lucretia is forceful or she is creative. If Lucretia is forceful, then she will be a good executive. It is not possible that Lucretia is both efficient and creative. If she is not efficient, then either she is forceful or she will be a good executive. Can we conclude that Lucretia will be a good executive?
9. How many Louisiana license plates can be formed given that they are unrestricted strings of 3 letters followed by 3 digits?
10. There are 10 balls in an urn; 4 are white, 6 are black. What is the probability that exactly one white ball or exactly one black ball is drawn in three draws, without replacement?
11. Let each number in the following puzzle represent a digit 0 through 9. Each digit may only be assigned to one letter. What is the value of each letter in the given alphametic if $S = 1$?
$$\begin{array}{r} \text{LAMA O} \\ + \text{STATE} \\ \hline \text{CONVT} \end{array}$$
12. What is the maximum number of non-overlapping triangles that can be formed with five straight lines?
13. What is the height of a cylinder with radius $\frac{\sqrt{\pi}}{\pi}$ whose volume is equal to that of a sphere with radius $\frac{1}{\pi}$?
14. "Fz jfb fz syq rfiz kj tfsyqtfsvuz bqjqb sk bqfrvsc, syqc fbq mks uqbsfvm; fz jfb fz syqc fbq uqbsfvm, syqc ok mks bqjqb sk bqfrvsc." –FREQBS QVMZSQVM
15. What is the determinant of the matrix $\begin{pmatrix} 1 & 1 & 3 & 4 \\ 2 & 0 & 6 & 2 \\ 3 & 3 & 1 & 4 \\ 1 & 0 & 3 & 1 \end{pmatrix}$?
16. The odometer of a car showed 15,951 miles. The driver noticed that this number is palindromic: it reads the same backward and forward. "Curious," the driver said to himself. "It will be a long time before that happens again." But 2 hours later, the odometer showed a palindromic number again. How fast was the car travelling?
17. A rectangular prism has a square base with a hole that is drilled through the entire length of the prism in the center of the square. If the square has a side length of 8 inches, the prism is 2 inches deep, and the diameter of the hole is 4 inches, what is the surface area of the surface?

18. How many zeroes are at the end of $\frac{131!}{60!}$?
19. Daniel is pulling socks from the dryer in, for whatever reason, pitch darkness. In the dryer he has 31 white socks, 18 grey socks, and 12 black socks. How many socks would he need to pull from the dryer in order to ensure that he has at least one pair of every color?
20. Esteban and Jorge are typing an article. Esteban types 30 words per minute, and Jorge types 70 words per minute. They each type one half of the article. In words per minute, what is the rate at which the article is typed?
21. What is the minimum number of yes/no questions needed to identify any space on a chessboard? (This question refers only to the space, not any piece that may occupy it.)
22. If a regular hexagon is inscribed inside of a circle with a radius of 1 unit, what is the area of the hexagon?
23. What is the decimal representation of $2120_3 + 41_5$?
24. A married couple has two children. The older child is a girl. What is the probability that both children are girls.
25. A married couple has two children. At least one of the two children is a boy. What is the probability that both children are boys?
26. Insert three signs (plus or minus) into the equation below to make it true:
- $$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9 = 100$$
27. A wanderer comes across a tome from ancient times with the following equations inscribed upon it:

$$(3\%6)=3 \qquad (7\&9)=63$$

$$(16\%12)=4 \qquad (4\&7)=28$$

$$(100\%5)=5 \qquad (4\&100)=100$$

Upon decoding, he stumbles across evidence that leads him to believe that the symbol “=” is being used to express equality. Using the above evidence, evaluate the following expression:

$$\left(\left((72\%72)\%6 \right)\%27 \right) \& \left(\left((50\%10)\&4 \right)\%8 \right)$$

28. A point E is chosen at random inside a square ABCD. What is the probability that angle AEB is acute?
29. Find the smallest positive integer x such that

$$x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{5}$$

$$x \equiv 6 \pmod{7}$$

That is, find the smallest positive integer x such that $2 = x - 3a$, $1 = x - 5b$, and $6 = x - 7c$ for some integral $a, b, c \geq 0$.

30. A circle is drawn with area 16π . A square is drawn inside of this circle such that the circle circumscribes the square. A smaller circle is then drawn inside of the square, and then a smaller square is drawn inside of the smaller circle. This process is repeated ad infinitum. What is the sum of the areas of the circles summed with the sum of the areas of the squares?