



Louisiana Mu Alpha Theta

affiliated with
Mu Alpha Theta
National High School and Junior College
Honorary Mathematics Club

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Mu A Integration

TEST RULES

1. Do not begin test until you are told to do so.
2. You must supply your own #2 pencil.
3. Only ACT approved calculators are allowed on all tests.
4. Print your name and school in the name blank, your code in the date blank, and the area test in the subject blank on your Scantron answer sheet
5. Standard procedure for machine graded papers must be followed. Use only a #2 pencil, marking the appropriate spaces carefully.
6. In case of a tie, winners will be determined according to the order in which the answer sheet was turned into the moderator.
7. Do all scratch work on your test.

For all problems, choice **E** is "None of the above".

- Evaluate $\int_0^4 \sqrt{16-x^2} dx$.
A. $\frac{\pi}{2}$ **B.** π **C.** 2π **D.** 4π **E.** NOTA
- Using four rectangles on a regular partition of $[0, 2]$, calculate the **lower sum** approximation of $\int_0^2 (x^2+1)dx$.
A. 1 **B.** $\frac{14}{3}$ **C.** $\frac{9}{4}$ **D.** $\frac{15}{4}$ **E.** NOTA
- If $\int_a^b f(x)dx=0$, then which of the following must be true?
A. $f(x)=0$ **B.** $a=b$ **C.** $f(-x)=-f(x)$
D. At least one of the choices **A**, **B**, or **C** **E.** NOTA
- Evaluate $\int \frac{x+e^x}{xe^x} dx$.
A. $-e^{-x} - \frac{1}{x^2} + C$ **B.** $e^{-x} - \ln|x| + C$ **C.** $e^{-x} + \ln|x| + C$
D. $-e^{-x} + \ln|x| + C$ **E.** NOTA
- Evaluate $\int \frac{2}{\sqrt{9-4x^2}} dx$.
A. $\sin^{-1}\left(\frac{2x}{3}\right) + C$ **B.** $\frac{1}{3}\sin^{-1}\left(\frac{2x}{3}\right) + C$ **C.** $\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right) + C$
D. $\sec^{-1}\left(\frac{2x}{3}\right) + C$ **E.** NOTA
- If $f(x)$ is a continuous function such that $\int_0^1 f(x)dx = -4$ and $\int_0^2 f(x)dx = 12$ then evaluate $\int_0^2 (3f(x)+6)dx$.
A. -12 **B.** 0 **C.** 48 **D.** 72 **E.** NOTA

7. Find the volume of a solid given that its base is an isosceles right triangle with legs of length four and cross sections perpendicular to one of its legs are semicircles.
- A. $\frac{\pi}{2}$ B. π C. $\frac{8\pi}{3}$ D. $\frac{16\pi}{3}$ E. NOTA
8. The velocity of a particle moving on a line at time t is $v(t) = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ feet per second. How many feet did the particle travel from $t = 1$ to $t = 9$ seconds?
- A. 536 B. 496 C. 492 D. 248 E. NOTA
9. What is the value of c guaranteed by the mean value theorem for integrals for the function $f(x) = \frac{6}{x^2}$ on the interval $[1, 2]$?
- A. $\sqrt{2}$ B. $2\sqrt{2}$ C. $\frac{3}{2}$ D. $\frac{\sqrt{2}}{2}$ E. NOTA
10. A particle moves along the x -axis. Find the average value of the velocity on the closed interval $[1, 4]$ when $v(t) = 2t^3 - 4t^2 + 3t + 2$.
- A. 22 B. 24 C. 33 D. 67 E. NOTA
11. If $F(x) = \int_0^{x^2} \frac{\tan(\pi t)}{(1+t)} dt$, find $F'\left(\frac{3}{2}\right)$.
- A. $\frac{12}{13}$ B. $\frac{8}{13}$ C. $\frac{4}{13}$ D. $\frac{3}{13}$ E. NOTA
12. What is the area of the region between the curves $y = 4\sin\left(\frac{x}{2}\right)$ and $y = 2\sin x$ on the interval $[0, 2\pi]$?
- A. 12 B. $12\sqrt{2}$ C. 12π D. $6\sqrt{3}$ E. NOTA
13. Solve for a : $\int_1^4 (4ax^2 + 2x + 3a) dx = \int_2^4 (2ax^2 - 3ax + 2) dx$.
- A. $\frac{3}{19}$ B. $\frac{5}{41}$ C. $-\frac{6}{55}$ D. $-\frac{7}{65}$ E. NOTA
14. What is the area of the region bounded by $f(x) = 10 - 2^x$ and $g(x) = 10$ on the interval $[0, 3]$?
- A. $7\ln 2$ B. $\frac{8}{\ln 2}$ C. $8\ln 2$ D. $\frac{7}{\ln 2}$ E. NOTA

15. Which of the following are antiderivatives of $\frac{\ln^2 x}{x}$?
- I. $\frac{\ln^3 x}{3}$ II. $\frac{\ln^3 x}{3} + 6$ III. $\frac{2 \ln x - \ln^2 x}{x^2}$
- A. I only B. III only C. I and II only
 D. I and III only E. NOTA
16. The region in the first quadrant bounded by the axes and the graphs of $\sqrt{x} + \sqrt{y} = 4$ and $x = 4$ is revolved about the x-axis. Find the volume of the solid generated (to the nearest hundredth).
- A. 764.04 B. 243.20 C. 92.15 D. 33.51
 E. NOTA
17. If $\int_1^{12} |cx - 6| dx = \frac{73}{3}$ where $1 < \frac{6}{c} < 12$, find c.
- A. $\frac{10}{3}$ B. $\frac{4}{3}$ C. $\frac{3}{2}$ D. $\frac{2}{3}$ E. NOTA
18. Find the general solution for the differential equation $(3x^2 + 9) \frac{dy}{dx} = xy$
- A. $y = C(3x^2 + 9)^{\frac{1}{6}}$ B. $y = C(x^2 + 3)^{\frac{1}{2}}$ C. $y = C(x^2 + 3)^{\frac{1}{3}}$
 D. $y = C(x^2 + 3)^{\frac{1}{6}}$ E. NOTA
19. Evaluate: $\int_1^e \left[\frac{1}{x} - \frac{1}{x} \cdot \ln\left(\frac{1}{x}\right) \right] dx$
- A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{2}$ D. $\frac{3}{2}$ E. NOTA
20. Solve the differential equation, $x^2 y' - x = 1$, satisfying the condition $y(1) = 2$.
- A. $y = 2 - \ln(x)$ B. $y = 2 - \ln(x^2)$ C. $y = 3 - \frac{1}{x} + \ln(x)$
 D. $y = 3 + \frac{1}{x} + \ln(x)$ E. NOTA

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Integration Test

21. Semi-circular cross sections parallel to the y-axis are taken along the graph of $y=3x^2-6x$ on the interval $[2, 4]$. Find the volume of the surface formed.

A. $\frac{186\pi}{5}$ B. $\frac{744\pi}{5}$ C. $\frac{1488\pi}{5}$ D. $\frac{372\pi}{5}$
 E. NOTA

22. If $f(x)$ is a continuous function such that $\int_1^9 f(x)dx=12$ and

$$\int_1^6 f(x)dx=15, \text{ then evaluate } \int_6^9 4(f(x)+2)dx.$$

A. -12 B. -6 C. 6 D. 12 E. NOTA

23. A particle moves along the x-axis so that its acceleration at any time t is given by $a(t)=6t-18$. At time $t=0$ the velocity of the particle is 24. At $t=1$ its position is 20. What is the total distance traveled by the particle from $t=0$ to $t=4$?

A. 4 B. 16 C. 20 D. 24 E. NOTA

24. For a certain curve, $\frac{dy}{dx}=\sqrt{3+xy+3x+y}$. The curve passes through the points $(-1, 1)$ and $(8, b)$. Find the value of b .

A. 8 B. 22 C. 118 D. 121 E. NOTA

25. Evaluate the indefinite integral: $\int \frac{2x}{x^2+6x+10} dx$

A. $\ln|x^2+6x+10|+C$ B. $2\ln|x^2+6x+10|+\arctan(x+3)+C$
 C. $\ln|x^2+6x+10|+6\arctan(x+3)+C$ D. $\ln|x^2+6x+10|-6\arctan(x+3)+C$
 E. NOTA

26. $F(x)=\int_0^{2\sin(x)} \sqrt{1+t^3} dt$. Find $F'(\pi)$.

A. -6 B. -2 C. 3 D. 6 E. NOTA

Tiebreaker: The area enclosed by the graphs of $y=x^2-4x$ and $y=x+6$ is rotated about the line $x=8$. Find the volume of the solid formed.