

Louisiana Mu Alpha Theta

affiliated with
Mu Alpha Theta
National High School and Junior College
Honorary Mathematics Club

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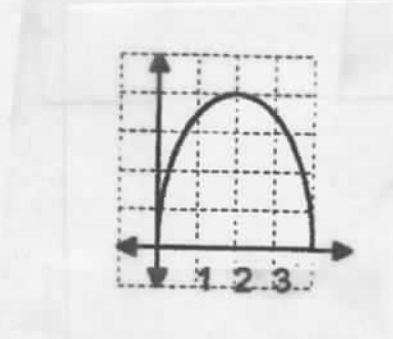
Mu A - Differentiation

TEST RULES

1. Do not begin test until you are told to do so.
2. You must supply your own #2 pencil.
3. Only ACT approved calculators are allowed on all tests.
4. Print your name and school in the name blank, your code in the date blank, and the area test in the subject blank on your Scantron answer sheet
5. Standard procedure for machine graded papers must be followed. Use only a #2 pencil, marking the appropriate spaces carefully.
6. In case of a tie, winners will be determined according to the order in which the answer sheet was turned into the moderator.
7. Do all scratch work on your test.

For all problems, choice **E** is "None of the above".

- Evaluate $\lim_{h \rightarrow 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$.
A. 0 **B.** $\frac{1}{2}$ **C.** 1 **D.** The limit does not exist
E. NOTA
- The sum of two nonnegative numbers, x and y , is 12. What is the largest possible product of x^2 and y ?
A. 0 **B.** 8 **C.** 36 **D.** 256 **E.** NOTA
- $f(x) = \arcsin(e^x)$. Find $f'(x)$.
A. $\frac{e^x}{\sqrt{1-e^{2x}}}$ **B.** $e^x \arccos(e^x)$ **C.** $\frac{e^x}{\sqrt{1-e^{2x}}}$
D. $-e^x \cos(e^x)$ **E.** NOTA
- At $x = 2$, the function given by $f(x) = \begin{cases} x^2, & x < 2 \\ 8 - 2x, & x \geq 2 \end{cases}$ is
A. differentiable but not continuous **B.** continuous but not differentiable
C. both continuous and differentiable
D. neither continuous nor differentiable **E.** NOTA
- $f(x) = 2^{\cot(x)}$. What is the slope of the normal to f at $x = \frac{\pi}{4}$?
A. $\frac{-1}{4 \ln 2}$ **B.** $\ln 16$ **C.** $\frac{-1}{8 \ln 2}$ **D.** $\frac{1}{\ln 16}$ **E.** NOTA
- The total cost of producing x units is given by $C(x) = 3000 - 20x + 0.03x^2$. If the total revenue is $R(x) = 50x - 0.01x^2$, find the marginal profit when $x = 100$ units.
A. 38 **B.** 62 **C.** 90 **D.** 162 **E.** NOTA
- If $f(x) = x^{17} + \sin(x)\cos(x) + e^x + 3$, what is $f'(1.1)$?
A. 0 **B.** 5.623 **C.** 11.077 **D.** 80.530
E. NOTA

8. A 100 ft long cable of diameter 4 inches is submerged in seawater, Because of corrosion, the surface area of the cable decreases at a rate of $750 \text{ in}^2 / \text{yr}$. Ignoring the corrosion at the ends of the cable, find the rate at which the diameter is decreasing in in/yr to the nearest hundredth?
A. 0.20 **B.** 0.31 **C.** 1.96 **D.** 2.39 **E.** NOTA
9. A metallic right circular cylinder is continuously heated and is expanding. If the height is 4 times the radius, what is the rate of change of the radius in inches/sec, when the volume is increasing at the rate of $24\pi \text{ in}^3 / \text{sec}$ and the radius is 1.5 inches?
A. $\frac{4}{9}$ **B.** $\frac{2}{9}$ **C.** $\frac{8}{9}$ **D.** $\frac{11}{9}$ **E.** NOTA
10. Find the maximum value of the rate of change of $f(x) = -3x^3 + 9x^2 + 52$.
A. 2 **B.** 9 **C.** 58 **D.** 109 **E.** NOTA
11. Given the graph of f , which of the following is true? f, f', f'' are all continuous and differentiable. Each tick mark represents a unit of 1.
A. $f(0) < f'(3) < f'(1)$
B. $f''(3) < f(0) < f'(1)$
C. $f'(2) < f'(3) < f'(1)$
D. $f(1) < f(0), f'(1)$ **E.** NOTA
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12. At what value of c is the rate of increase of $\sqrt[3]{c}$ three times the rate of the increase of c ?
A. $\frac{\sqrt{3}}{27}$ **B.** $\frac{\sqrt[3]{3}}{3}$ **C.** $\frac{\sqrt[3]{3}}{9}$ **D.** $\frac{\sqrt{3}}{9}$ **E.** NOTA
13. If $f(x) = 3^{\sin x}$, find $f'\left(\frac{\pi}{6}\right)$.
A. $(3)^{\frac{\sqrt{3}}{2}}$ **B.** $1.5 \ln \sqrt{3}$ **C.** 3 **D.** $3 \ln \sqrt{3}$ **E.** NOTA
14. $f(x) = \sqrt{1 + 2 \sin(x)}$. Use the linearization of f at $x = 0$ to approximate $f(0.03)$.
A. -1.015 **B.** -0.97 **C.** 1 **D.** 1.03
E. NOTA

15. While delivering his presents on Christmas Eve, Santa, who is 6 feet tall, not the 20 feet some kids think, was walking toward a house that had a light 24 feet above the ground. If he was magically walking toward the house at the rate of 36 ft/sec, (he did have a lot of distance to cover in a limited amount of time.), what was the rate of change of the length of his shadow in ft/sec when he was 21 feet from the house?

A. -12 **B.** -4 **C.** 4 **D.** 12 **E.** NOTA

16. If $f'(x) = \frac{f(x)}{x+3}$ and $f(0) = 2$, find $f'(8)$.

A. $\frac{2}{3}$ **B.** $\frac{4}{3}$ **C.** $\frac{21}{3}$ **D.** $\frac{25}{3}$ **E.** NOTA

17. For what values of m and b will the following function have a derivative for every x ?

$$f(x) = \begin{cases} x^2 + x - 3, & x \leq 1 \\ mx + b & x > 1 \end{cases}$$

A. $m = 3, b = -4$ **B.** $m = -2, b = -3$ **C.** $m = 1, b = -4$
D. $m = -2, b = 1$ **E.** NOTA

18. Woodstock is on top of a loblolly pine 80 feet above a lake watching Snoopy in a motorboat moving directly away from the base of the tree at a rate of 25 feet per second. How fast is the angle of depression of Woodstock's line of sight changing when the boat is 150 feet from the base of the tree?

A. -0.08 rad/sec **B.** -0.03 rad/sec **C.** -0.05 rad/sec
D. -0.07 rad/sec **E.** NOTA

19. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = x^3 + k$, for every value of k ?

A. $y = -3x^2$ **B.** $y = \frac{-\ln x^2}{3}$ **C.** $y = \frac{1}{3x}$ **D.** $y = \frac{-1}{x^3}$
E. NOTA

20. If $\frac{dy}{dt} = f'(t)$ and $\frac{dy}{dx} = h(t)$, then $\frac{d^2y}{dx^2} =$

A. $\frac{h''(t)}{f'(t)}$ **B.** $\frac{h'(t)}{f''(t)}$ **C.** $\frac{h'(t)}{f'(t)}$ **D.** $f'(t) \cdot h'(t)$ **E.** NOTA

21. The graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $x = -1$. Find k .
- A.** 1 **B.** -1 **C.** 2 **D.** -2 **E.** NOTA

22. Use Newton's method to calculate x_2 in approximating the zero for $y = \sin(x) - e^{x-3}$ with an initial guess of $x_0 = 1.5$. (Round your answer to the nearest thousandth)
- A.** 2.248 **B.** 2.587 **C.** 4.447 **D.** 5.562
E. NOTA

23. Using the table of values for the function f , what is the best estimate for $f'(1.8)$?

x	1	1.4	1.6	1.8	2.0	2.2
$F(x)$	3	3.9	3.7	4.2	4.7	4.4

- A.** 0.4 **B.** 2.5 **C.** 1.4 **D.** 0.5 **E.** NOTA
24. If $f(x) = x^3 + x^2 + x - 1$, find the derivative of $f^{-1}(x)$ where $x = 2$ on the graph of $f^{-1}(x)$.
- A.** 6 **B.** $\frac{1}{5}$ **C.** $\frac{1}{6}$ **D.** $\frac{1}{17}$ **E.** NOTA
25. Find $\frac{d}{dx}(x \sin^{-1} x + \sqrt{1-x^2})$
- A.** $\sin^{-1} x$ **B.** $\frac{1}{1-x^2}$ **C.** $\frac{x}{1-x^2}$ **D.** 0
E. NOTA
26. A tank with rectangular base and rectangular sides is to be open at the top. It is to be constructed so that the width of its base is 6 meters and its volume is 48 cubic meters. If building the tank costs \$12 per square meter for the base and \$8 per square meter for the sides, what is the cost of the least expensive tank to the nearest cent?
- A.** \$428.30 **B.** \$450.70 **C.** \$532.50
D. \$598.30 **E.** NOTA

Tiebreaker: Let $f(x) = x + \sin^2 x$ for $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$. Find the **exact** maximum value of f on the given interval.